A NEW METHOD OF MEASURING THE ACCELERATION OF GRAVITY AT SEA

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Introduction.—The desirability of more extensive and more accurate measurements of the acceleration of gravity at sea has recently been emphasized by Bauer[1], Bowie[2], and Hayford.[3] Such measurements are essential in the accurate determination of the figure of the earth. Ocean gravity measurements now available are due almost wholly to Hecker,[4] who employed the mercurial barometer-hypsometer method in an elaborate and extended series of measurements in the Atlantic, Pacific, and Indian oceans. In this method the outstanding difference at a given station between the atmospheric pressure as computed from the boiling point of water and directly observed with the mercurial barometer is attributed to the difference in the gravitational force acting on the mercurial column at the given station and at the standard station (Lat. 45°, sea level).

There are certain difficulties inherent in the barometer-hypsometer method which greatly lessen its usefulness. (1) The atmospheric pressure must be determined in absolute measure by each method in order that the determinations may be comparable, so that systematic errors are serious; (2) the boiling point determinations must be carried out with the highest degree of refinement in order to secure even moderate precision in the determination of g. The observed boiling points are not simply differential measurements. The true temperature interval between the melting point of ice and the observed boiling point must be known in terms of the hydrogen-scale before reference can be made to vapor pressure tables for the determination of the atmospheric pressure.

An independent measure of the accuracy attainable in the determination of atmospheric pressure by the boiling point method is afforded by measurements of the 'fundamental interval' of standard mercurial thermometers. Waidner and Dickinson[5] found in a study of the standard mercurial thermometers of the Bureau of Standards that the fundamental intervals varied through a range of 0.015C. during the ten-day period covered by the measurements. This variation they attribute in part to the sticking of the meniscus with resulting variation in the capillary pressure.

The probable error of a fundamental interval determination in Waid-

ner and Dickinson's measurements is about ±0°003C. The measurements were carried out with a refinement which is probably unattainable at sea, and may be taken to represent the limit of accuracy attainable in such determinations on board ship. We may now consider the effect of such an error in the determination of g. A probable error of ±0°003C. in the true boiling point temperature would correspond in atmospheric pressure to ±0.083 mm. of mercury and to a probable error of ±0.11 cm. per sec. per sec. in the value of g. The uncertainties due to the boiling-point measurements alone under the most favorable conditions would, therefore, result in a probable error of 1 part in 10,000 in the value of g. This is 13 times the limit of error set by Hayford for ocean gravity measurements. To this must be added the errors in the barometric observations. Errors in the boiling-point temperature alone would justify Bowie's statement that Hecker's gravity measurements at sea "are subject to uncertainties as large as the largest of the new-method anomalies of gravity in the United States, that is, between 0.05 dyne and 0.10 dyne."

FIG. 1.

Description of the New Method.—The new method of measuring g at sea described in this paper is based upon the change in height of a barometric column sealed off from communication with the atmosphere.[6] All boiling-point determinations are thus avoided. The apparatus is kept at constant temperature in a bath of melting ice. The determination only of the position of the upper end of the barometric column is necessary. The design of the instrument is such that in making this setting the enclosed gas mass is automatically reduced to a constant volume; and since temperature is constant, the measurements are made at constant pressure.

A sketch of the glass part of the apparatus is shown in figure 1. The mercurial column is contained in the capillary c (bore 0.6–0.7 mm.) the lower end of which opens beneath mercury in the bottom of the gas chamber d. This capillary is sealed to the wall of the gas-chamber where it passes through the upper end. The upper part of the capillary is bent into a flexible zig-zag, and expands into the spherical bulb b (diameter 2 cm.) The bulb contains a fixed iron point p sealed to the inside of the
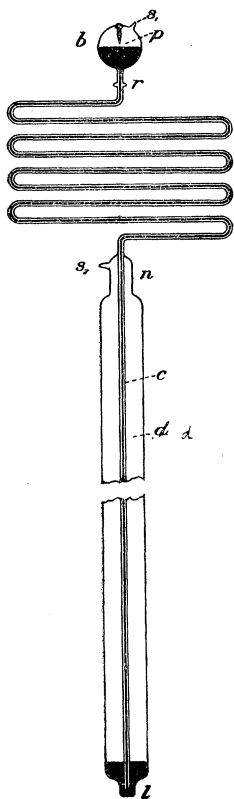
bulb by means of an inserted platinum wire and extending vertically downward, so that the point is approximately at the center of the bulb. The length of the mercurial column is about 74 cm.

The flexible capillary permits a slight vertical movement of the observing bulb with respect to the, gas chamber. This movement is determined by a micrometer screw of 1 mm. pitch which controls the motion of a carriage in which the observing bulb is rigidly mounted. The carriage slides on parallel rods mounted on a base which is rigidly cemented to the neck n of the gas chamber, so that the position of the bulb relative to the gas chamber is definitely determined by the screw.

After the apparatus has been thoroughly cleaned and dried, pure mercury is introduced in excess of the amount required to fill the capillary and observing bulb. The apparatus is then highly evacuated through s_1 and s_2 and the observing bulb sealed at s_1. Dry nitrogen is introduced into the gas chamber in excess of atmospheric pressure through a stop-cock temporarily sealed to the chamber at s_2. The micrometer head is then mounted, the protective casing of the gas chamber adjusted, and the whole apparatus placed in the ice-tank in a vertical position and surrounded with melting ice. A large U-type vacuum manometer is connected with the stop-cock of the gas chamber and the air exhausted from the connecting tube. After temperature equilibrium is attained, the stop-cock of the gas chamber is opened and the pressure of the nitrogen slowly reduced until the mercury stands in contact with the fixed point in the observing bulb when the carriage is near the middle of its range. The pressure in the gas chamber at this time is determined by reading the difference in level of the mercury surfaces in the manometer by means of a cathetometer. The observed pressure, corrected to mercury at 0°C., gives the difference in height of the two mercury surfaces in the gravity apparatus. The length of the column, less the scale reading, gives the 'constant' of the instrument, which is added to subsequent micrometer readings to obtain the true height of the mercurial column.

When the above determination is made, the gas chamber connection is sealed off with a blow-pipe. The apparatus is now perfectly gas-tight, since it is entirely free from ground joints, stop-cocks and sealed-in platinum connections, and the original openings at s_1 and s_2 have both been sealed with the blow-pipe. The apparatus is also readily transportable, since at ordinary temperatures the pressure in the gas chamber is sufficient to fill the observing bulb with mercury. It can then be tipped in any position.

In observing, the apparatus is supported in a vertical position in

the cork-covered ice tank and completely covered with fragments of melting ice 1 to 3 cm. in thickness, dropped loosely into the tank. The fixed point is observed through a glass tube introduced through the ice and is illuminated through a similar tube on the opposite side. The observing tube is equipped with a low power lens. The ice tank is 25 cm. in diameter and is covered with cork lagging 7.5 cm. thick.

At sea the tank is swung from gimbals, the outer gimbal ring being suspended from 4 spiral steel springs hung from the four corners of a frame of 1-inch galvanized pipe, the four legs of which are screwed to the deck. Movable weights on the bottom of the tank serve to adjust the apparatus to a vertical position, which is determined by means of a sensitive level on the head of the instrument. When the apparatus is mounted on the open deck, an effective wind shield is essential, the supports of which must be entirely independent of the apparatus.

Theory.—Let us determine through a series of observations the height of the mercurial column at some station where g is accurately known. Let us now consider the apparatus to be transported to another station, where g is greater. The mercurial column will be depressed, compressing the gas. The observing bulb is accordingly lowered until the fixed point is again in grazing contact with the mercury surface. The volume of mercury in the bulb and capillary is now the same as at the first station, since the slight flexure of the capillary produces no appreciable change in its volume. Therefore the volume of mercury in the gas chamber is also the same as in the first observation. In other words, the volume of the gas is constant when the upper mercury surface is in grazing contact with the fixed point, and is independent of the position of the bulb. Since the temperature is constant, the pressure p of the gas is always the same at the time of taking an observation. We have then

$$p = \rho g h = g_1 h_1 \qquad (1)$$

in which g and g_1 represent the acceleration of gravity at the two stations, h and h_1 the corresponding heights of the mercury column, and ρ the density of mercury. The latter is constant, since the mercury is always at the temperature of melting ice. Equation (1) then reduces to

$$g / g_1 = h_1 / h \qquad (2)$$

or, the height of the column is inversely proportional to the acceleration of gravity. If the height h is measured at some station where g is known, the acceleration of gravity at any other station may therefore be determined simply by measuring h_1.

Let $h - h_1 = \Delta h$ represent the difference in the height of the column at the two stations. On substituting this value of h_1 in equation (2) we have

$$g \ / \ g_1 = 1 - \Delta h \ / \ h \tag{3}$$

in which Δh represents simply the difference of the micrometer readings at the two stations, and does not involve the absolute height of the column. Since h is at least 200 times as large as Δh, the absolute height of the column does not, in this method, need to be determined with great precision.

Discussion of errors.—The apparatus was designed with the object of obtaining, if possible, an accuracy of 1 part in 100,000 in the measurements. The discussion of the various sources of error will therefore be made on this basis.

Temperature variation.—During the measurements the whole apparatus was surrounded with melting ice, so that the effect of slight variations in temperature need be considered only in connection with the temperature of the gas. Artificial ice was employed and all ice that was not perfectly clear and crystalline was discarded. The purity was systematically checked by measuring the electrical conductivity of the tap water. The impurities were never sufficient to depress the theoretical freezing point more than $0°.001$C., whereas a variation of $\pm 0°.003$C. would be necessary to produce a change of 1 in 100,000 in the gas pressure. The drip water was allowed to escape from the ice tank through a small trap near the bottom in order to insure the ice extending below the bottom of the gas chamber.[7]

Error in setting point in contact with mercury surface.—Lord Rayleigh[8] found in his investigations with the micromanometer that the fixed ground glass point could be set in contact with the mercury surface with an error not exceeding ± 0.0015 mm. An accuracy of 1 part in 100,000 in gravity observations necessitates a probable error in the micrometer observations not greater than ± 0.008 mm., which is readily attainable from a series of readings at sea under favorable conditions. The writer has found that in measurements at sea a metallic point is superior to a glass point, due to the fact that the latter becomes electrified through the motion of the mercury surface and this influences the readings.

Error in the determination of the instrument constant.—Reference to equation (3) will show that since the height of the column is at least 200 times the total range (Δh), the uncertainty in h may be 100 times that permissible in Δh. The constant of the instrument (i.e., the vertical distance from the lower mercury surface to the zero on the scale)

therefore does not need to be known with an accuracy greater than 0.5 mm. Determinationsof the constant agreeing to 0.1 mm. can be obtained with the auxiliary manometer to which reference has already been made.

Thermal hysteresis of the glass envelop.—This phenomenon is generally recognized in precision thermometry, and results in the so-called depression of the zero. This effect would tend to introduce an uncertainty in the determination of the difference in the acceleration of gravity at two stations where it is necessary to remove the apparatus from the ice in proceeding from one station to the other. A slight change of this kind (2 parts in 100,000) was observed at Balboa, following the dismantling of the apparatus and its transportation across the isthmus by rail, owing to the slides in the canal. These instruments were made of German glass. Waidner and Dickinson[9] state that thermometers made of the best boro-silicate glass show a depression of the zero of 0°.03. This would correspond to a change of 1 part in 200,000 in the volume of the bulb. Consequently if boro-silicate glass were used in the construction of the glass parts of the apparatus, the error due to thermal hysteresis would fall within the limit set in this discussion.

Disturbances arising from the motion of the ship.—Three classes of disturbances are encountered on board ship:

1. *Tremors*, due to the engines or auxiliary machinery or to the impact of waves. The effect of such disturbances can be greatly reduced by suspending the ice tank from spiral springs. This form of support does not eliminate all vibration. Slight tremors however appear to be advantageous rather than otherwise, as they help to bring the mercury surface to its true position in the bulb.

2. *Horizontal translation*, due chiefly to rolling, which tends to swing the apparatus from its vertical position. Such translations produce two effects which can be made to counteract each other to some extent. (a) The vertical component of the column is shortened by an amount proportional to the sine of the deflection from the vertical. Such deflections therefore tend to increase the length of the column. (b) The centrifugal force resulting from the deflection tends to depress the column when the latter is mainly below the point of support. This can be eliminated by mounting the column so that it is bisected by the gimbal plane. By suitably adjusting the position of the column, the two effects can thus be made to compensate in part.[10]

3. *Vertical motion*, due to rolling and pitching or to the rise and fall of the ship as a whole in a heavy sea. This is the most serious of all the disturbances to contend with, for the motion is accelerated, and is superimposed on the gravitational acceleration. Extensive damping through the use of the capillary column has been employed in the apparatus here described to reduce

the effect of vertical motion. The expansion of the top of the column into a bulb having a cross-sectional area 500–800 times that of the capillary reduces proportionally the change in level in comparison with the actual movement of mercury in the capillary. The rate of change in the acceleration of gravity with latitude is so slight that extensive damping is permissible from this standpoint, although the time required to secure an observation is of course correspondingly increased.

Correction for the course and speed of the ship.—Eötvös[11] has shown the necessity of applying a correction for the easterly or westerly motion of the ship, due to the fact that the ship's motion modifies the angular velocity of revolution of the apparatus about the earth's axis. The centrifugal force acting on the mercurial column when on board a ship moving east or west is therefore not the same as when the ship is at rest or moving north or south. The correction may be as great as 1 part in 10,000, but can be accurately computed if the course, speed, and approximate latitude of the ship are known.

The probable error of the observations.—In 1914, observations were made from Sydney, Australia, to San Francisco by way of Wellington, N. Z.; and in 1915, two instruments were taken from New York to San Francisco via Panama. The following table shows the results of gravity determinations on board ship in various harbors during the two voyages, and where pendulum observations are available near the stations they are appended for comparison. The 1915 observations show a mean probable error in the harbor determinations of 13 parts in 1,000,000. The readings of the two instruments at sea were for the most part as consistent as at the harbor stations. The ocean measurements will be discussed in a later paper.

The method is by no means to be considered as perfected. One of the instruments used in 1915 gave results consistently lower than the other. This indicates a systematic error which must be located. It is also highly desirable that new instruments constructed of boro-silicate glass should be carried over the same course several times with different sea conditions in order to determine whether systematic errors are introduced into measurements made on a rough sea.

Acknowledgments.—In the first apparatus constructed, the gas chamber and capillary were made of steel. This apparatus developed a leak on the voyage to Sydney, and repeated attempts to repair it in the Fiji Islands and at Sydney met with failure. This disappointment was however more than offset by the kindness of Dr. J. A. Pollock, Professor of Physics in the University of Sydney, who placed the facilities of his laboratory and the services of his glass-blower and mechanician

TABLE I.

GRAVITY DETERMINATIONS ON BOARD SHIP AT HARBOR STATIONS

1914

With San Francisco as base,

$\dfrac{\text{cm.}}{\text{(sec.)}^2}$

g at Wellington, N. Z.,

observed, instrument No. 1	980.316
by pendulum (Wright)	980.292

+0.024

g at Sydney, N. S. W.,

observed, instrument No. 1	979.68
by pendulum (Smithsonian tables)	979.69

−0.01

1915

With New York as base,

g at Colon, Panama,

observed, instrument No. 2	978.196
instrument No. 5	978.236

978.216±0.013

g at Balboa, Panama,

observed, instrument No. 2	978.175
instrument No. 5	978.195

978.185±0.007

g at San Francisco,

observed, instrument No. 2	979.940
instrument No. 5	979.995

979.968±0.019

by pendulum (Smithsonian tables)	979.98

0.012±0.019

at my disposal in the construction of a new apparatus of glass, which was used on the return voyage.

I am also deeply indebted to Dr. C. G. Abbot, Director of the Astrophysical Observatory of the Smithsonian Institution, for his assistance in connection with the steel apparatus, and for valued suggestions regarding the construction of the later form.

Lieut. R. S. Wright, R.E., has kindly supplied the value of g at Wellington from his unpublished pendulum observations made in connection with the Scott Antarctic expedition.

The voyage in 1914 was made with grants from the Australian and New Zealand governments, in connection with the Australian meeting of the British Association for the Advancement of Science. I desire to express my obligation to these governments and also to the American Associ-

ation for the Advancement of Science for a grant used in connection with the 1915 measurements.

[1] Bauer, L. A., On gravity determinations at sea, *Amer. J. Sci.*, **31**, 1 (1911). Hecker's remarks on ocean gravity observations, *Amer. J. Sci.* **33**, 245 (1912).

[2] Bowie, Wm., Isostasy and the size and shape of the earth, *Science*, **39**, 705 (1914).

[3] Hayford, J. F., these PROCEEDINGS, **2**, 394 (1916).

[4] For a description of the apparatus employed by Hecker and a summary of his ocean measurements, see Hecker, O., Bestimmung der Schwerkraft auf dem Schwarzen Meere und, an dessen Kuste sowie neue Ausgleichung der Schwerkraftmessungen aud dem Atlantischen, Indischen, und Groszen Ozean, *Zentralbur. Internat. Erdmessung, Veroffentlichungen Berlin*, N. F., Nr. 20 (1910).

[5] *Washington, D. C., Bull. Bur. Standards*, **3**, 663 (1907).

[6] This principle has already been employed by Mascart, [*Paris, C. R. Acad. Sci.*, **95**, 631, 1882], who used a sealed-off barometer of the U-tube type. The temperature was not controlled, and volume and pressure were both variable. The gas volume had therefore to be measured, the pressure corresponding to the observed temperature and volume calculated, and finally reduced to standard conditions.

[7] This procedure now seems undesirable, since Pernet has observed a departure of 0°.01C. from the true zero when the bulb of a thermometer is surrounded by artificial ice freely drained, due to the ice being undercooled. (See Pernet, J., Sur les moyens d'eliminer l'influence de la variation des points fixes des thermometres a mercure. *Travaux. Bur., International des Poids et Mesures*, 1881, second partie.) In the writer's measurements, ice was added in small quantities at frequent intervals so as to keep the tanks completely filled, and remained in the apparatus for several hours before melting sufficiently to sink to the level of the gas chamber. It is consequently doubtful whether the effect observed by Pernet influenced the measurements, but it is a possible source of error which can be avoided by keeping the interstitial spaces filled with water.

[8] *Phil. Trans. R. Soc., London*, **196**, 205 (1901).

[9] *Loc. cit.*

[10] The above procedure was not followed strictly in the trials of the apparatus which have so far been made at sea, due to the difficulty of following the observing tube with the eye when the ship is rolling in a heavy sea. Experience shows however that such an arrangement is necessary if observations are to be made in rough weather, and the difficulty in observing can apparently be met by a modification of the viewing apparatus.

[11] See Hecker, *loq. cit.*

THE PROBLEM OF CONTINENTAL FRACTURING AND DIASTROPHISM IN OCEANICA

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Paleogeographic studies during the past thirty years have been developing the hypothesis that the ancient continental platforms were arranged latitudinally rather than longitudinally as they are now, and, further, that their areal extent, including their emergent and submerged portions, was greater than at present. It appears that vast land-masses have been fractured, broken up, and more or less permanently taken